

**DIPLOMA EXAMINATION IN ENGINEERING/TECHNOLOGY/
MANAGEMENT/COMMERCIAL PRACTICE, APRIL – 2023**

ENGINEERING MATHEMATICS – I

[Maximum Marks: 100]

[Time: 3 Hours]

PART-A

[Maximum Marks: 10]

I. (Answer *all* questions in one or two sentences. Each question carries 2 marks)

1. Find the value of $\tan 150^\circ$.
2. If $\tan \alpha = \frac{12}{5}$ and α is an acute angle, Find $\cos \alpha$.
3. Find the area of triangle having $a=4, b=2, C = 30^\circ$.
4. Evaluate $\lim_{x \rightarrow 3} \frac{x^3-27}{x-3}$
5. Find the slope of $y = x^{3/2}$ at $x = 1$. (5 x 2 = 10)

PART-B

[Maximum Marks: 30]

II. (Answer any *five* of the following questions. Each question carries 6 marks)

1. Prove that $\cos(A + B) \cos(A - B) = \cos^2 A - \sin^2 B$.
2. Prove that $\sin 120^\circ \cos 330^\circ + \cos 240^\circ \sin 330^\circ = 1$.
3. Prove that $R(a^2 + b^2 + c^2) = abc(\cot A + \cot B + \cot C)$.
4. The sides of a triangle are in the ratio 4:5:6. Find the angles of the triangle.
5. Using the quotient rule find the derivative of $\tan x$.
6. If $y = x^2 \cos x$, prove that $x^2 \frac{d^2 y}{dx^2} - 4x \frac{dy}{dx} + (x^2 + 6)y = 0$
7. Prove that a rectangle of fixed perimeter has its maximum area when it becomes a square. (5 x 6 = 30)

PART-C

[Maximum Marks: 60]

(Answer **one** full question from each Unit. Each full question carries **15** marks)

UNIT - I

III. (a) Prove that $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A.$ (5)

(b) If $\sin A = 3/5$, $\cos B = 12/13$ and A and B are acute angles, find $\tan(A + B).$ (5)

(c) Express $\sqrt{3} \cos x + \sin x$ in the form $R \sin(x + \alpha).$ (5)

OR

IV. (a) Prove that $\frac{\sin \theta}{1+\cos \theta} + \frac{1+\cos \theta}{\sin \theta} = 2 \operatorname{cosec} \theta.$ (5)

(b) If $A+B = 45^\circ$, show that $(1+\tan A)(1+\tan B) = 2.$ (5)

(c) A light house is 20 metres high. An observer on the top of the light house observes a boat at an angle of depression 30° . How far is the boat from the observer? (5)

UNIT - II

V. (a) Prove that $1 + \tan \theta \tan 2\theta = \sec 2\theta.$ (5)

(b) Show that $\cos 55^\circ + \cos 65^\circ + \cos 175^\circ = 0.$ (5)

(c) Show that $a(b \cos C - c \cos B) = b^2 - c^2.$ (5)

OR

VI. (a) Prove that $\frac{\sin 3A}{\sin A} - \frac{\cos 3A}{\cos A} = 2.$ (5)

(b) Show that $\sin 10^\circ \sin 50^\circ \sin 70^\circ = 1/8.$ (5)

(c) Solve the triangle, given $a = 87\text{cm}$, $b=53\text{cm}$ and $C=70^\circ.$ (5)

UNIT- III

VII. (a) Evaluate $\lim_{x \rightarrow \infty} \frac{3x^2 - 5x + 9}{2x^2 + 4x + 7}$ (5)

(b) Find $\frac{dy}{dx}$ if i) $y = x^2 \sin^{-1} x$ ii) $y = \log \sin x.$ (3+2=5)

(c) If $y = \sin x \cos x$, show that $\frac{d^2y}{dx^2} + 4y = 0.$ (5)

OR

- VIII. (a) Evaluate i) $\lim_{x \rightarrow 0} \frac{2x^2 + 3x}{3x^2 - 4x}$ ii) $\lim_{x \rightarrow 0} \frac{\sin 3x}{x}$. (3+2=5)
- (b) Using first principles, find the derivative of \sqrt{x} . (5)
- (c) If $y = \tan^{-1}x$, prove that $(1 + x^2) y'' + 2xy' = 0$. (5)

UNIT - IV

- IX. (a) Find the equation of the tangent and normal to the semi circle
 $y = \sqrt{25 - x^2}$ at the point (4,3). (5)
- (b) The distance travelled by a particle moving along a straight line after t time is given by
 $s = 2t^3 - 9t^2 + 12t + 6$. find the value of t when acceleration is zero. (5)
- (c) Find the maximum value of $4x^3 + 9x^2 - 12x + 2$ (5)

OR

- X. (a) Prove that the function $x^3 - 3x^2 + 6x + 7$ is an increasing function for all real values of x . (5)
- (b) A spherical balloon is inflated by pumping 25cc of gas per second. Find the rate at which the radius of the balloon is increasing when its radius is 15cms. (5)
- (c) The deflection of a beam is given by $y = 2x^3 - 9x^2 + 12x$. find the maximum deflection. (5)
